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Transient Analysis of Anisotropic Multilayered Media Subjected to Dynamic Antiplane Loadings

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The transient responses of anisotropic multilayered media subjected to arbitrarily distributed antiplane loadings are investigated. To solve the complicated problem, a linear coordinate transformation is introduced and successfully used to transform the anisotropic layered medium problem to an isotropic problem. The relationship between field quantities of the anisotropic problem and the corresponding isotropic problem are established for a Cartesian coordinate system. The boundary value problem is solved by using the integral transform method. The solutions in the Laplace transform domain are constructed in the form of a power series of the phase-related reflection and transmission matrices. Each term in the series represents a reflected or transmitted wave. The transient solution is then obtained by means of Cagniard's method. The corresponding static solution is also derived by application of the final value theorem. For numerical calculations, the transient responses of an anisotropic thin layer overlying a half-space are considered. The transition behavior from transient response to static value is presented and discussed in detail.

Nomenclature

$A^{(i)}$, $B^{(i)}$	=	field coefficients
C_{ij}	=	elastic moduli
$\boldsymbol{c}$	=	global field vector
$\boldsymbol{M}$	=	coefficient matrix
$\boldsymbol{p}$	=	source vector
$\boldsymbol{q}$	=	response vector
$\boldsymbol{R}$	=	global phase-related reflection and transmission matrix
$\boldsymbol{R}_{cv}$	=	phase-related receiver matrix
t	=	time coordinate
$\boldsymbol{t}$	=	global boundary displacement-traction vector
$\boldsymbol{t}^{[i]}$	=	applied displacement-traction vector
W	=	antiplane displacement in corresponding isotropic multilayered media
w	=	antiplane displacement in anisotropic multilayered media
X, Y, Z	=	rectangular coordinates in corresponding isotropic multilayered media
x, y, z	=	rectangular coordinates in anisotropic multilayered media
μ	=	shear modulus
ρ	=	material mass density
τ_{XZ}, τ_{YZ}	=	shear stresses in corresponding isotropic multilayered media
τ_{xz}, τ_{yz}	=	shear stresses in anisotropic multilayered media

Introduction

FOR the past two decades, the importance of composite materials has increased very rapidly because of their high strength and light weight. Because of the fast development in material science, anisotropic materials have been widely used in these composite structures for modern engineering applications. These anisotropic laminated composites may be subjected to static or dynamic loadings. They have already received quite widespread attention in the form of static analyses in the past. However, the study of dynamic responses of anisotropic composite laminates is rare in the literature. Thus, a better understanding of how the transient responses of anisotropic composites subjected to dynamic loadings transit to static fields is very important.

The investigation of interaction between transient waves and boundaries was initialized by Lamb.[1] He studied the problem of an isotropic half-space subjected to dynamic point and line loads on the surface. Since then, many researchers, such as Garvin[2] and Tsai and Ma,[3] devoted their efforts to the study of isotropic half-space problems. For layered media problems, Thomson[4] and Haskell[5] developed a matrix method to determine the dispersion relation for elastic waves in isotropic multilayered media. Later, Gilbert and Backus[6] proposed the propagator matrix and gave a more formal mathematical interpretation to the technique. Ma and Huang[7] investigated a composite with n isotropic layers subjected to dynamic antiplane impact loading on the surface. They derived the transfer relation expressed for the general representations of the response between each layer and used Cagniard's method[8] to obtain the transient solution in the time domain. After the appearance of Cagniard's method, Spencer[9] proposed the generalized ray method to analyze the surface response of an isotropic layered half-space to the radiation from a localized source. Later, the theory of a generalized ray was used to solve many transient problems by Müller,[10,11] Ceranoglu and Pao,[12] and Borejko.[13] Recently, the transient responses of isotropic multilayered media subjected to dynamic antiplane and in-plane loadings were studied analytically by Lee and Ma.[14,15] The connection between the matrix method and the generalized ray method is established by considering the reflection and transmission of waves at a single interface. The corresponding numerical calculation and experimental measurement for an isotropic layered half-space subjected to dynamic in-plane loading were presented by Ma and Lee.[16] Following the same method, Ma et al.[17] reexamined the transient problem of an isotropic multilayered medium subjected to arbitrarily antiplane loadings. They illustrated numerical examples of an isotropic layered half-space and investigated the characteristic time of transition from transient responses to static values in detail.

Studies of the effects of anisotropy on elastic waves were started in the 1950s due to applications in seismology and crystal physics. For anisotropic solids, many researchers were already devoting time to the study of elastodynamic problems: Kraut,[18] Burridge,[19] Tewary and Fortunko,[20] Budreck,[21] and so forth. Recently, Ting[22] pointed out that there have been several new developments in the theory and applications of anisotropic elasticity. Ting[23] also reviewed recent developments in anisotropic elasticity. Because of the mathematical complexity, most of investigations on elastodynamics of anisotropic laminated media were carried out by numerical methods by, for example, Nelson and Dong,[24] Dong and Huang,[25] Kausel,[26]

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Datta et al.,[27] Seale and Kausel,[28] Liu et al.,[29,30] and Liu and Achenbach.[31,32] Thus far, the analytical studies of transient responses of anisotropic multilayered media have been limited.

In this study, the transient responses of anisotropic multilayered media subjected to dynamic antiplane loadings are investigated analytically. A linear coordinate transformation proposed by Lin and Ma[33] is used in this study to simplify the anisotropic problem. Lin and Ma[33] have used this methodology to solve the static problem for anisotropic multilayered media. This study extends this transformation method to solve more complicated transient problems. The linear coordinate transformation reduces the anisotropic multilayered problem to an equivalent isotropic problem with a similar geometry configuration. A matrix method proposed by Lee and Ma[14] is then used to solve the corresponding isotropic boundary value problem directly. The solution in the Laplace transform domain is obtained by expanding the matrix solution into a power series of the phase-related reflection and transmission matrix. The inverse transform is carried out by the application of Cagniard's method.[8] The transient solutions for shear stresses in a time domain are expressed in a closed form. Each term in the formulation has its own physical meaning. The solutions are valid for an infinite length of time and have accounted for the contributions of an infinite number of reflected waves. Static solutions for shear stresses are also obtained by the use of the final value theorem. Numerical results are evaluated for the special case of an anisotropic layered half-space problem. The transition behavior from transient responses to static values is investigated and discussed in detail.

Linear Coordinate Transformation

Consider an antiplane deformation in an anisotropic medium. Given the absence of body force, the two-dimensional antiplane wave motion of a homogeneous, anisotropic, and linearly elastic solid is governed by

$$C_{55}\frac{\partial^2 w(x,y,t)}{\partial x^2} + 2C_{45}\frac{\partial^2 w(x,y,t)}{\partial x \partial y} + C_{44}\frac{\partial^2 w(x,y,t)}{\partial y^2} = \rho\frac{\partial^2 w(x,y,t)}{\partial t^2} \tag{1}$$

where $w(x,y,t)$ is the antiplane displacement in the z direction; C_{ij}, $i,j = 4,5$, are elastic moduli; and ρ is the mass density of the anisotropic material. The xy plane has been assumed to coincide with one of the planes of material symmetry such that in-plane and antiplane deformations are uncoupled. The relevant stress components are

$$\tau_{yz}(x,y,t) = C_{44}\frac{\partial w(x,y,t)}{\partial y} + C_{45}\frac{\partial w(x,y,t)}{\partial x} \tag{2}$$

$$\tau_{xz}(x,y,t) = C_{45}\frac{\partial w(x,y,t)}{\partial y} + C_{55}\frac{\partial w(x,y,t)}{\partial x} \tag{3}$$

Introduce a linear coordinate transformation[34–36]:

$$X = x - (C_{45}/C_{44})y \tag{4}$$

$$Y = (C_e/C_{44})y \tag{5}$$

$$Z = z \tag{6}$$

where

$$C_e = \sqrt{C_{44}C_{55} - C_{45}^2} \tag{7}$$

It is assumed that C_{44} and C_{45}, as well as $\sqrt{(C_{44}C_{55} - C_{45}^2)}$, are all positive. The transformation given by Eqs. (4–6) reduces Eq. (1) to the standard wave equation in the (X, Y) coordinate system as

$$\frac{\partial^2 W(X,Y,t)}{\partial X^2} + \frac{\partial^2 W(X,Y,t)}{\partial Y^2} = b^2\frac{\partial^2 W(X,Y,t)}{\partial t^2} \tag{8}$$

where $W(X,Y,t)$ is the displacement in the Z direction and

$$W(X,Y,t) = w(x,y,t) \tag{9}$$

$$b = \sqrt{C_{44}\rho}\big/C_e \tag{10}$$

It is easy to verify from Eqs. (2) and (3) that the relevant stress components in the anisotropic solid are related to those in the corresponding isotropic solid by

$$\tau_{YZ}(X,Y,t) = C_e\frac{\partial W(X,Y,t)}{\partial Y} \tag{11}$$

$$\tau_{XZ}(X,Y,t) = C_e\frac{\partial W(X,Y,t)}{\partial X} \tag{12}$$

$$\tau_{yz}(x,y,t) = \tau_{YZ}(X,Y,t) \tag{13}$$

$$\tau_{xz}(x,y,t) = \frac{C_{45}}{C_{44}}\tau_{YZ}(X,Y,t) + \frac{C_e}{C_{44}}\tau_{XZ}(X,Y,t) \tag{14}$$

From Eqs. (8), (11), and (12), note that the original anisotropic problem is converted into an equivalent isotropic problem by setting $C_e = \mu$ (shear modulus). From the relationship of displacement and shear stresses for an anisotropic solid and the corresponding isotropic solid expressed in Eqs. (9), (13), and (14), note that it is possible to obtain the solution for an anisotropic problem from a corresponding result of an isotropic problem.

Transient Solutions for Anisotropic Multilayered Media in Double Transform Domain

In this section, the transient response of anisotropic multilayered media subjected to arbitrarily distributed antiplane loadings is analyzed. Consider an infinitely anisotropic medium consisting of n layers separated by parallel planes, as shown in Fig. 1. Each layer is made of an anisotropic, homogeneous, and linearly elastic material. It is assumed that the interfaces are perfectly bounded. All quantities related to the ith layer are followed by a superscript (i). The two-dimensional governing equation for antiplane deformation in each layer can be expressed as

$$C_{55}^{(i)}\frac{\partial^2 w^{(i)}(x,y,t)}{\partial x^2} + 2C_{45}^{(i)}\frac{\partial^2 w^{(i)}(x,y,t)}{\partial x \partial y} + C_{44}^{(i)}\frac{\partial^2 w^{(i)}(x,y,t)}{\partial y^2} = \rho^{(i)}\frac{\partial^2 w^{(i)}(x,y,t)}{\partial t^2}, \qquad i = 1,2,\ldots,n \tag{15}$$

The boundary conditions on the top and bottom surfaces of the layered medium are limited to the traction type, and so the boundary conditions on these two surfaces can be written as

$$\tau_{yz}^{(0)}(x,0,t) = \tau_{yz}^{[0]}(x,t) \tag{16}$$

for $-\infty < x < \infty$, and

$$\tau_{yz}^{(n)}(x,-h_n,t) = \tau_{yz}^{[n]}(x,t) \tag{17}$$

for $-\infty < x < \infty$, where

$$h_n = \sum_{i=1}^{n} h^{(i)} \tag{18}$$

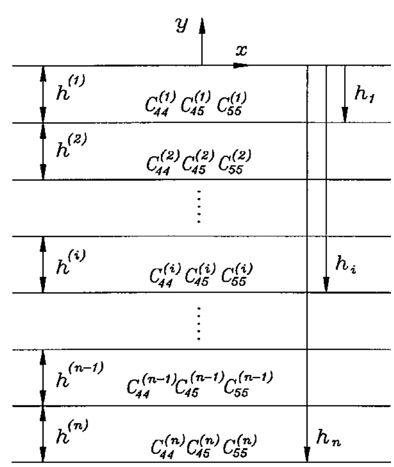

Fig. 1 Configuration and coordinate system of an anisotropic n-layered medium.

in which $h^{(i)}$ is the thickness of the ith layer, and $\tau^{[i]}$ is the specified traction. The loading applied at the interface between two adjacent layers can be either a displacement or traction and yields the following boundary conditions:

$$w^{(i)}(x,-h_i,t)-w^{(i+1)}(x,-h_i,t)=w^{[i]}(x,t) \qquad i=1,2,\ldots,n-1 \quad (19)$$

$$\tau_{yz}^{(i)}(x,-h_i,t)-\tau_{yz}^{(i+1)}(x,-h_i,t)=\tau_{yz}^{[i]}(x,t) \qquad i=1,2,\ldots,n-1 \quad (20)$$

where $w^{[i]}$ is the specified displacement.

The transient problem of the anisotropic multilayered medium can be treated as a boundary value problem as indicated in Eqs. (15–20). It is difficult to solve this boundary value problem directly because of the presence of many material constants. It is desirable to reduce the dependence on material constants in advance of the analysis of a given problem. A linear coordinate transformation introduced in the preceding section is used to simplify the governing wave equation. To maintain the geometric continuity of the layered configuration, the linear coordinate transformation expressed in Eqs. (4–6) are modified for each layer, as follows[33]:

$$X=x+\alpha^{(i)}y+\sum_{k=1}^{i-1}h_k\left(\alpha^{(k)}-\alpha^{(k+1)}\right), \qquad i=1,2,\ldots,n \quad (21)$$

$$Y=\beta^{(i)}y+\sum_{k=1}^{i-1}h_k\left(\beta^{(k)}-\beta^{(k+1)}\right), \qquad i=1,2,\ldots,n \quad (22)$$

$$Z=z, \qquad i=1,2,\ldots,n \quad (23)$$

where

$$\alpha^{(i)}=-C_{45}^{(i)}\big/C_{44}^{(i)} \quad (24)$$

$$\beta^{(i)}=C_e^{(i)}\big/C_{44}^{(i)} \quad (25)$$

$$C_e^{(i)}=\sqrt{C_{44}^{(i)}C_{55}^{(i)}-C_{45}^{(i)}} \quad (26)$$

When Eqs. (21–23) are applied to Eq. (15), the governing equations in the transformed (X,Y) coordinate system can be reduced to standard wave equations, as follows:

$$\frac{\partial^2W^{(i)}(X,Y,t)}{\partial X^2}+\frac{\partial^2W^{(i)}(X,Y,t)}{\partial Y^2}=\left(b^{(i)}\right)^2\frac{\partial^2W^{(i)}(X,Y,t)}{\partial t^2} \qquad i=1,2,\ldots,n \quad (27)$$

where

$$W^{(i)}(X,Y,t)=w^{(i)}(x,y,t) \quad (28)$$

$$b^{(i)}=\sqrt{C_{44}^{(i)}\rho^{(i)}\big/\left(C_e^{(i)}\right)^2} \quad (29)$$

The stress–displacement relations expressed in the (X,Y) coordinate system within each layer become

$$\tau_{YZ}^{(i)}=C_e^{(i)}\frac{\partial W^{(i)}(X,Y,t)}{\partial Y}, \qquad i=1,2,\ldots,n \quad (30)$$

$$\tau_{XZ}^{(i)}=C_e^{(i)}\frac{\partial W^{(i)}(X,Y,t)}{\partial X}, \qquad i=1,2,\ldots,n \quad (31)$$

The relevant stress components in the anisotropic solid are related to those in the corresponding isotropic solid by

$$\tau_{yz}^{(i)}(x,y,t)=\tau_{YZ}^{(i)}(X,Y,t) \quad (32)$$

$$\tau_{xz}^{(i)}(x,y,t)=\beta^{(i)}\tau_{XZ}^{(i)}(X,Y,t)-\alpha^{(i)}\tau_{YZ}^{(i)}(X,Y,t) \quad (33)$$

The boundary conditions and the jump conditions expressed in Eqs. (16–20) can be represented as follows:

$$\tau_{YZ}^{(0)}(X,0,t)=\tau_{YZ}^{[0]}(X,t) \quad (34)$$

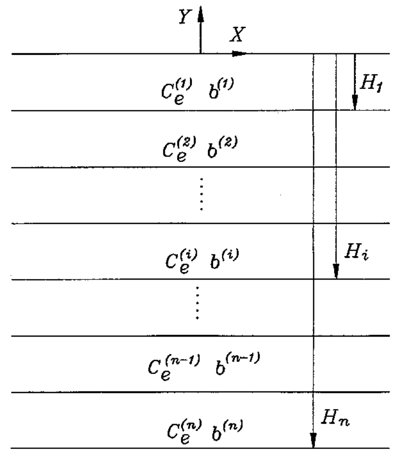

Fig. 2 Configuration and coordinate system of the n-layered medium after the linear coordinate transformation.

for $-\infty<X<\infty$,

$$\tau_{YZ}^{(n)}(X,-H_n,t)=\tau_{YZ}^{[n]}(X,t) \quad (35)$$

for $-\infty<X<\infty$,

$$W^{(i)}(X,-H_i,t)-W^{(i+1)}(X,-H_i,t)=W^{[i]}(X,t) \qquad i=1,2,\ldots,n-1 \quad (36)$$

$$\tau_{YZ}^{(i)}(X,-H_i,t)-\tau_{YZ}^{(i+1)}(X,-H_i,t)=\tau_{YZ}^{[i]}(X,t) \qquad i=1,2,\ldots,n-1 \quad (37)$$

where

$$H_n=\beta^{(n)}h_n+\sum_{k=1}^{n-1}(\beta_k-\beta_{k+1})h_k \quad (38)$$

Note that the governing equations in Eq. (27) and constitutive equations in Eqs. (30) and (31) are similar to the isotropic case. Moreover, note that no gaps or overlaps are generated along the interfaces under the modified linear coordinate transformation. The new geometric configuration in the transformed (X,Y) coordinate system is shown in Fig. 2. It is found that the thickness of each layer and the location of the applied loading are both changed. However, the interfaces between two adjacent layers remain continuous and are parallel to the X axis. Thus, the new geometric configuration is similar to the original problem. This means that the modified linear coordinate transformation successfully changes the original anisotropic multilayered problem to the corresponding isotropic multilayered problem with a similar geometric configuration and boundary conditions. Consequently, once the boundary value problem described in Eqs. (27), (30), (31), and (34–37) can be solved, the solutions for the original anisotropic problem can be obtained from Eqs. (28), (32), and (33).

The boundary value problem just mentioned can be solved by the application of integral transform methods. Lee and Ma[14] have used an effective matrix method to solve the isotropic layered medium problem. A similar procedure will be performed in the following derivation. The one-sided Laplace transform with respect to time and the two-sided Laplace transform with respect to X are defined by[37]

$$\bar f(X,Y,s)=\int_0^\infty f(X,Y,t)e^{-st}\,\mathrm{d}t \quad (39)$$

$$\bar f^*(\lambda,Y,s)=\int_{-\infty}^{\infty}\bar f(X,Y,s)e^{-s\lambda X}\,\mathrm{d}X \quad (40)$$

When the one-sided Laplace transform is applied over time t and the two-sided Laplace transform over X under the restriction of

$\mathrm{Re}(\gamma_T^{(i)}) > 0$, Eq. (27) becomes an ordinary differential equation with the general solution, as follows:

$$\bar{W}^{*(i)}(Y, \lambda, s) = A^{(i)}(\lambda, s)e^{s\gamma_T^{(i)}Y} + B^{(i)}(\lambda, s)e^{-s\gamma_T^{(i)}Y} \tag{41}$$

where

$$\gamma_T^{(i)} = \left(b^{(i)2} - \lambda^2\right)^{\frac{1}{2}} \tag{42}$$

$A^{(i)}$ and $B^{(i)}$ are field coefficients of each layer and denote the downgoing and upgoing waves, respectively.

By substituting Eq. (41) into the stress–displacement relation in Eq. (30), we can obtain

$$\begin{pmatrix} \bar{W}^{*(i)} \\ \bar{\tau}_{YZ}^{*(i)} \end{pmatrix} = \begin{pmatrix} M_{11}^{(i)} & M_{12}^{(i)} \\ M_{21}^{(i)} & M_{22}^{(i)} \end{pmatrix} \begin{pmatrix} A^{(i)} \\ B^{(i)} \end{pmatrix} \tag{43}$$

where the phase-related receiver elements are

$$M_{11}^{(i)}(Y; \lambda, s) = e^{s\gamma_T^{(i)}Y} \tag{44}$$

$$M_{12}^{(i)}(Y; \lambda, s) = e^{-s\gamma_T^{(i)}Y} \tag{45}$$

$$M_{21}^{(i)}(Y; \lambda, s) = sC_e^{(i)}\gamma_T^{(i)}e^{s\gamma_T^{(i)}Y} \tag{46}$$

$$M_{22}^{(i)}(Y; \lambda, s) = -sC_e^{(i)}\gamma_T^{(i)}e^{-s\gamma_T^{(i)}Y} \tag{47}$$

When the double integral transform is applied to the boundary conditions (34–37) and then substituted into Eq. (43), the field coefficients $A^{(i)}$ and $B^{(i)}$ for each layer can be determined and expressed as follows:

$$\begin{pmatrix} M_{21}^{(1)}(0) & M_{22}^{(1)}(0) \end{pmatrix} \begin{pmatrix} A^{(1)} \\ B^{(1)} \end{pmatrix} = \bar{\tau}_{YZ}^{*[0]} = \bar{t}^{*[0]} \tag{48}$$

at top surface $Y = 0$,

$$\begin{pmatrix} M_{21}^{(n)}(-H_n) & M_{22}^{(n)}(-H_n) \end{pmatrix} \begin{pmatrix} A^{(n)} \\ B^{(n)} \end{pmatrix} = \bar{\tau}_{YZ}^{*[n]} = \bar{t}^{*[n]} \tag{49}$$

at bottom surface $Y = -H_n$, and

$$\begin{bmatrix} M_{11}^{(i)}(-H_i) & M_{12}^{(i)}(-H_i) & -M_{11}^{(i+1)}(-H_i) & -M_{12}^{(i+1)}(-H_i) \\ M_{21}^{(i)}(-H_i) & M_{22}^{(i)}(-H_i) & -M_{21}^{(i+1)}(-H_i) & -M_{22}^{(i+1)}(-H_i) \end{bmatrix} \begin{bmatrix} A^{(i)} \\ B^{(i)} \\ A^{(i+1)} \\ B^{(i+1)} \end{bmatrix} = \begin{bmatrix} \bar{W}^{*[i]} \\ \bar{\tau}_{YZ}^{*[i]} \end{bmatrix} = \bar{t}^{*[i]} \tag{50}$$

at interface $Y = -H_i$, $i = 1, 2, \ldots, n-1$. Here, $\bar{t}^{*[i]}$ is the transformed field for applied displacement–traction vector $t^{[i]}$, $i = 0, 1, \ldots, n$, and

$$t^{[0]} = \tau_{YZ}^{[0]} \tag{51}$$

$$t^{[n]} = \tau_{YZ}^{[n]} \tag{52}$$

$$t^{[i]} = \left(W^{[i]} \quad \tau_{YZ}^{[i]}\right)^T, \qquad i = 1, 2, \ldots, n-1 \tag{53}$$

When a global field vector c with $2n$ elements is defined as

$$c(\lambda, s) = \left[\begin{pmatrix} A^{(1)}(\lambda, s) \\ B^{(1)}(\lambda, s) \end{pmatrix}^T \quad \begin{pmatrix} A^{(2)}(\lambda, s) \\ B^{(2)}(\lambda, s) \end{pmatrix}^T \quad \cdots \quad \begin{pmatrix} A^{(n)}(\lambda, s) \\ B^{(n)}(\lambda, s) \end{pmatrix}^T\right]^T \tag{54}$$

and when the applied displacement–traction vectors $t^{[i]}$, $i = 0, 1, \ldots, n$, are combined into a global boundary displacement–traction vector as

$$\bar{t}^*(\lambda, s) = \left(\bar{t}^{*[0]^T} \quad \bar{t}^{*[1]^T} \quad \cdots \quad \bar{t}^{*[n-1]^T} \quad \bar{t}^{*[n]^T}\right)^T \tag{55}$$

then the system of equations expressed in Eqs. (48–50) can be rewritten in a compact matrix form:

$$Mc = \bar{t}^* \tag{56}$$

Here, the coefficient matrix M is a $2n \times 2n$ matrix given by

$$M = D + L + U = \begin{bmatrix} D_0 & U_0 & & & & & \\ L_1 & D_1 & U_1 & & & & \\ & L_2 & D_2 & \ddots & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \ddots & D_{n-2} & U_{n-2} & \\ & & & & L_{n-1} & D_{n-1} & U_{n-1} \\ & & & & & L_n & D_n \end{bmatrix} \tag{57}$$

in which D represents the diagonal block matrix. U and L indicate the nonzero block elements of upper and lower triangular matrices, respectively. Their corresponding elements are defined as follows:

$$D_0 = M_{21}^{(1)}(0) \tag{58}$$

$$D_i = \begin{bmatrix} M_{12}^{(i)}(-H_i) & -M_{11}^{(i+1)}(-H_i) \\ M_{22}^{(i)}(-H_i) & -M_{21}^{(i+1)}(-H_i) \end{bmatrix} \qquad i = 1, 2, \ldots, n-1 \tag{59}$$

$$D_n = M_{22}^{(n)}(-H_n) \tag{60}$$

$$U_0 = \begin{bmatrix} M_{22}^{(1)}(0) & 0 \end{bmatrix} \tag{61}$$

$$U_i = \begin{bmatrix} -M_{12}^{(i+1)}(-H_i) & 0 \\ -M_{22}^{(i+1)}(-H_i) & 0 \end{bmatrix}, \qquad i = 1, 2, \ldots, n-2 \tag{62}$$

$$U_{n-1} = \begin{bmatrix} -M_{12}^{(n)}(-H_{n-1}) \\ -M_{22}^{(n)}(-H_{n-1}) \end{bmatrix} \tag{63}$$

$$L_1 = \begin{bmatrix} M_{11}^{(1)}(-H_1) \\ M_{21}^{(1)}(-H_1) \end{bmatrix} \tag{64}$$

$$L_i = \begin{bmatrix} 0 & M_{11}^{(i)}(-H_i) \\ 0 & M_{21}^{(i)}(-H_i) \end{bmatrix}, \qquad i = 2, 3, \ldots, n-1 \tag{65}$$

$$L_n = \begin{bmatrix} 0 & M_{21}^{(n)}(-H_n) \end{bmatrix} \tag{66}$$

Because the diagonal block matrix D is nonsingular, the global field vector c in Eq. (56) can be solved directly by

$$c = M^{-1}\bar{t}^* \tag{67}$$

The coefficient matrix M can be represented in an alternative form as

$$M = D(I + D^{-1}L + D^{-1}U) = D(I - R) \tag{68}$$

where

$$\boldsymbol{R} = -\boldsymbol{D}^{-1}(\boldsymbol{L}+\boldsymbol{U}) = \begin{bmatrix} 0 & -\boldsymbol{D}_0^{-1}\boldsymbol{U}_0 & & & & & \\ -\boldsymbol{D}_1^{-1}\boldsymbol{L}_1 & 0_{2\times 2} & -\boldsymbol{D}_1^{-1}\boldsymbol{U}_1 & & & & \\ & -\boldsymbol{D}_2^{-1}\boldsymbol{L}_2 & 0_{2\times 2} & \ddots & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & \ddots & 0_{2\times 2} & -\boldsymbol{D}_{n-2}^{-1}\boldsymbol{U}_{n-2} & \\ & & & & -\boldsymbol{D}_{n-1}^{-1}\boldsymbol{L}_{n-1} & 0_{2\times 2} & -\boldsymbol{D}_{n-1}^{-1}\boldsymbol{U}_{n-1} \\ & & & & & -\boldsymbol{D}_n^{-1}\boldsymbol{L}_n & 0 \end{bmatrix} \tag{69}$$

Therefore, the global field vector $\boldsymbol{c}$ in Eq. (67) can be expressed by

$$\boldsymbol{c} = (\boldsymbol{I}-\boldsymbol{R})^{-1}\boldsymbol{p} \tag{70}$$

where the source vector $\boldsymbol{p}$ that represents source waves generated by applied loadings is defined by

$$\boldsymbol{p} = \boldsymbol{D}^{-1}\bar{\boldsymbol{t}}^{*} = \begin{bmatrix} \boldsymbol{D}_0^{-1}\bar{\boldsymbol{t}}^{*[0]} \\ \boldsymbol{D}_1^{-1}\bar{\boldsymbol{t}}^{*[1]} \\ \vdots \\ \boldsymbol{D}_{n-1}^{-1}\bar{\boldsymbol{t}}^{*[n-1]} \\ \boldsymbol{D}_n^{-1}\bar{\boldsymbol{t}}^{*[n]} \end{bmatrix} \tag{71}$$

When the continuity conditions are applied at the interfaces, the elements $-\boldsymbol{D}_i^{-1}\boldsymbol{L}_i$ and $-\boldsymbol{D}_i^{-1}\boldsymbol{U}_i$, $i=0,1,\ldots,n$, in Eqs. (69) can be obtained as follows[14]:

$$-\boldsymbol{D}_i^{-1}\boldsymbol{L}_i = \begin{bmatrix} 0 & R_{i/i+1} \\ 0 & T_{i/i+1} \end{bmatrix} \tag{71}$$

$$-\boldsymbol{D}_i^{-1}\boldsymbol{U}_i = \begin{bmatrix} T_{i+1/i} & 0 \\ R_{i+1/i} & 0 \end{bmatrix} \tag{72}$$

where the phase-related reflection and transmission coefficients are

$$R_{i/i+1}(\lambda,s) = r_{i/i+1}\exp\left(-2s\gamma_T^{(i)}H_i\right) \tag{73}$$

$$T_{i/i+1}(\lambda,s) = t_{i/i+1}\exp\left[-s\left(\gamma_T^{(i)}-\gamma_T^{(i+1)}\right)H_i\right] \tag{74}$$

$$R_{i+1/i}(\lambda,s) = r_{i+1/i}\exp\left(2s\gamma_T^{(i+1)}H_i\right) \tag{75}$$

$$T_{i+1/i}(\lambda,s) = t_{i+1/i}\exp\left[s\left(\gamma_T^{(i+1)}-\gamma_T^{(i)}\right)H_i\right] \tag{76}$$

in which $r_{i/i+1}$ and $t_{i/i+1}$ (or $r_{i+1/i}$ and $t_{i+1/i}$) are the reflection and transmission coefficients, respectively, and

$$r_{i/i+1}(\lambda) = \frac{C_e^{(i)}\gamma_T^{(i)}(\lambda) - C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda)}{C_e^{(i)}\gamma_T^{(i)}(\lambda) + C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda)} \tag{77}$$

$$t_{i/i+1}(\lambda) = \frac{2C_e^{(i)}\gamma_T^{(i)}(\lambda)}{C_e^{(i)}\gamma_T^{(i)}(\lambda) + C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda)} \tag{78}$$

$$r_{i+1/i}(\lambda) = \frac{C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda) - C_e^{(i)}\gamma_T^{(i)}(\lambda)}{C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda) + C_e^{(i)}\gamma_T^{(i)}(\lambda)} \tag{79}$$

$$t_{i+1/i}(\lambda) = \frac{2C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda)}{C_e^{(i)}\gamma_T^{(i)}(\lambda) + C_e^{(i+1)}\gamma_T^{(i+1)}(\lambda)} \tag{80}$$

Hence, the global phase-related reflection and transmission matrix $\boldsymbol{R}$ expressed in Eq. (69) can be rewritten in terms of the local reflection and transmission coefficients, as follows:

$$\boldsymbol{R}(\lambda,s) = \left[\begin{array}{cc:cc:cc:cc} 0 & R_{1/0} & & & & & & \\ R_{1/2} & 0 & 0 & T_{2/1} & & & & \\ \hdashline T_{1/2} & 0 & 0 & R_{2/1} & & & & \\ & & R_{2/3} & 0 & 0 & T_{3/2} & & \\ \hdashline & & T_{2/3} & 0 & 0 & R_{3/2} & & \\ & & & \ddots & \ddots & \ddots & \ddots & \\ \hdashline & & & \ddots & \ddots & \ddots & \ddots & \\ & & & & R_{n-1/n} & 0 & 0 & T_{n/n-1} \\ \hdashline & & & & T_{n-1/n} & 0 & 0 & R_{n/n-1} \\ & & & & 0 & 0 & R_{n/n+1} & 0 \end{array}\right] \tag{81}$$

From Eqs. (43), (54), and (70), note that once the global field vector $\boldsymbol{c}$ is solved, the response functions $\bar{W}^{*(i)}$ and $\bar{\tau}_{YZ}^{*(i)}$ in the double transform domain for each layer can be obtained immediately. When a response vector

$$\boldsymbol{q}(Y,\lambda,s) = \left[\begin{pmatrix} \bar{W}^{*(1)}(Y,\lambda,s) \\ \bar{\tau}_{YZ}^{*(1)}(Y,\lambda,s) \end{pmatrix}^T \begin{pmatrix} \bar{W}^{*(2)}(Y,\lambda,s) \\ \bar{\tau}_{YZ}^{*(2)}(Y,\lambda,s) \end{pmatrix}^T \cdots \begin{pmatrix} \bar{W}^{*(n)}(Y,\lambda,s) \\ \bar{\tau}_{YZ}^{*(n)}(Y,\lambda,s) \end{pmatrix}^T\right]^T \tag{82}$$

is defined, then the global field can be related to the response vector by

$$\boldsymbol{q} = \boldsymbol{R}_{cv}(\boldsymbol{I}-\boldsymbol{R})^{-1}\boldsymbol{p} \tag{83}$$

where $\boldsymbol{R}_{cv}$ is the phase-related receiver matrix and is given by

$$\boldsymbol{R}_{cv}(Y,\lambda,s) = \left[\begin{array}{cc:cc:c:cc} M_{11}^{(1)} & M_{12}^{(1)} & & & & & \\ M_{21}^{(1)} & M_{22}^{(1)} & & & & & \\ \hdashline & & M_{11}^{(2)} & M_{12}^{(2)} & & & \\ & & M_{21}^{(2)} & M_{22}^{(2)} & & & \\ \hdashline & & & & \ddots & & \\ \hdashline & & & & & M_{11}^{(n)} & M_{12}^{(n)} \\ & & & & & M_{21}^{(n)} & M_{22}^{(n)} \end{array}\right] \tag{84}$$

By expansion of the matrix $(\boldsymbol{I}-\boldsymbol{R})^{-1}$ into power matrix series of $\boldsymbol{R}$, Eq. (83) can be rewritten as

$$\boldsymbol{q}=\boldsymbol{R}_{cv}\sum_{i=0}^{\infty}\boldsymbol{R}^{i}\boldsymbol{p} \tag{85}$$

where

$$\sum_{i=0}^{\infty}\boldsymbol{R}^{i}=(\boldsymbol{I}-\boldsymbol{R})^{-1} \tag{86}$$

It can be verified that the term $\boldsymbol{R}^{i}\boldsymbol{p}$ represents a group of waves that is reflected or transmitted through the interface i times.[14] The matrix series is convergent, and the expansion is suitable for practical problems because only finite terms have to be calculated during the transient process.

For the time being, the sources in the preceding derivation are limited to dynamic loadings applied at the interfaces. Now we consider the response of the layered medium subjected to dynamic loadings that are located within layers. The incident field induced by the loading within a layer can be separated into upgoing and downgoing waves. The source waves propagating in two directions will meet the interfaces at a later time. It is envisaged that the succeeding reflected and transmitted waves can be regarded as new sources at the interfaces. Thus, the solution can be deduced by modifying the source vector $\boldsymbol{p}$ expressed in Eq. (71). The new sources can be obtained by multiplying the matrix $\boldsymbol{R}$ to the modified source vector $\boldsymbol{p}^{*}$. The original source term should be considered individually and will be denoted by a vector $\boldsymbol{p}_0^{*}$. For instance, the solution in double transform domain for a body source applied at $Y=-H_{pi}$ in the ith layer can be represented as follows:

$$\boldsymbol{q}=\boldsymbol{R}_{cv}\boldsymbol{p}_0^{*}+\boldsymbol{R}_{cv}\sum_{i=1}^{\infty}\boldsymbol{R}^{i}\boldsymbol{p}^{*} \tag{87}$$

where

$$\boldsymbol{p}_0^{*}(\lambda,s)=\left(0,0,\cdots,0,\boldsymbol{p}_{H_{pi}}^{(i)},\cdots,0\right)^{T} \tag{88}$$

for $-H_{i-1}>Y>-H_{pi}>-H_i$,

$$\boldsymbol{p}_0^{*}(\lambda,s)=\left(0,0,\cdots,\boldsymbol{p}_{-H_{pi}}^{(i)},0,\cdots,0\right)^{T} \tag{89}$$

for $-H_{i-1}>-H_{pi}>Y>-H_i$, and

$$\boldsymbol{p}^{*}(\lambda,s)=\left(0,0,\cdots,p_{-H_{pi}}^{(i)},\boldsymbol{p}_{H_{pi}}^{(i)},\cdots,0,0\right)^{T} \tag{90}$$

for all Y.

Note that the summation index in the matrix series expressed in Eq. (87) begins with $i=1$ because the original source term has been separated from the summation. Thus far, the solution for the corresponding isotropic layered medium subjected to a dynamic body source is successfully derived and expressed in Eq. (87). The transient solution can be obtained by Cagniard's method[8] of Laplace inversion. Note that the solutions shown in Eq. (87) are expressed in the transformed (X,Y) coordinate system. The corresponding solutions for the anisotropic multilayered medium in the (x,y) coordinate system can be obtained from Eqs. (28), (32), and (33), in which the coordinate relations expressed in Eqs. (21–23) are used.

Transient and Static Solutions for an Anisotropic Layered Half-Space

In this section, an anisotropic layered half-space subjected to a dynamic antiplane concentrated load at the top layer is considered. The thin layer and half-space are both made of a homogeneous, anisotropic, and linearly elastic material. The thickness of the top

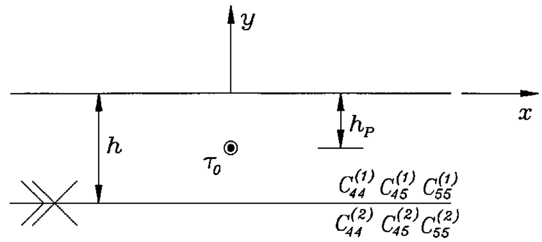

Fig. 3 Configuration and coordinate system of an anisotropic layered half-space.

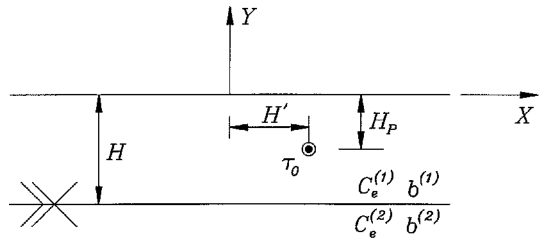

Fig. 4 Configuration and coordinate system for the layered half-space after the linear coordinate transformation.

layer is h, and the geometric configuration and coordinate system are shown in Fig. 3. For time $t<0$, the thin-layered structure is stress free and at rest. At time $t=0$, a dynamic antiplane concentrated load with magnitude τ_0 is applied suddenly at $(x,y)=(0,-h_P)$ within the top layer. The time dependence of the loading is represented by the Heaviside step function $H(t)$. By the use of the linear coordinate transformation proposed in Eqs. (21–23) (by setting $n=2$), the location of the applied load is shifted to $(X,Y)=(H',-H_P)$ in the transformed coordinate system, as shown in Fig. 4. Because the half-space is semi-infinite, the global field vector has only three elements, that is, $\boldsymbol{c}=(A^{(1)}\;\;B^{(1)}\;\;A^{(2)})^{T}$. The source vectors can be represented as follows:

$$\boldsymbol{p}_0^{*}=\left[\frac{\tau_0}{2s^2C_e^{(1)}\gamma_T^{(1)}}\exp\left(s\gamma_T^{(1)}H_P-s\lambda H'\right)\quad 0\quad 0\right]^{T} \tag{91}$$

for $0>-H_P>Y>-H$,

$$\boldsymbol{p}_0^{*}=\left[0\quad\frac{\tau_0}{2s^2C_e^{(1)}\gamma_T^{(1)}}\exp\left(-2\gamma_T^{(1)}H_P-s\lambda H'\right)\quad 0\right]^{T} \tag{92}$$

for $0>Y>-H_P>-H$, and

$$\boldsymbol{p}^{*}=\left[\frac{\tau_0}{2s^2C_e^{(1)}\gamma_T^{(1)}}\exp\left(s\gamma_T^{(1)}H_P-s\lambda H'\right)\quad\frac{\tau_0}{2s^2C_e^{(1)}\gamma_T^{(1)}}\exp\left(-2s\gamma_T^{(1)}H_P-s\lambda H'\right)\quad 0\right]^{T} \tag{93}$$

for all Y. The phase-related reflection and transmission matrix $\boldsymbol{R}$ is a 3×3 matrix and has the following form:

$$\boldsymbol{R}(\lambda,s)=\begin{bmatrix}0 & r_{1/0} & 0\\ r_{1/2}e^{-s2\gamma_T^{(1)}H} & 0 & 0\\ t_{1/2}e^{-s(\gamma_T^{(1)}-\gamma_T^{(2)})H} & 0 & 0\end{bmatrix} \tag{94}$$

where the reflection coefficient $r_{1/0}$ at free surface $Y=0$ is unity, and $r_{1/2}$ and $t_{1/2}$ are the reflection coefficient in Eq. (77) and the transmission coefficient in Eq. (78), respectively.

If the responses of the top layer are considered, the response vector is a 2×1 matrix given by

$$\boldsymbol{q}=\left[\bar{W}^{*(1)}\quad\bar{\tau}_{YZ}^{*(1)}\right]^{T} \tag{95}$$

and the receiver matrix is

$$\boldsymbol{R}_{cv} = \begin{bmatrix} e^{s\gamma_T^{(1)}Y} & e^{-s\gamma_T^{(1)}Y} & 0 \\ sC_e^{(1)}\gamma_T^{(1)}e^{s\gamma_T^{(1)}Y} & -sC_e^{(1)}\gamma_T^{(1)}e^{-s\gamma_T^{(1)}Y} & 0 \end{bmatrix} \tag{96}$$

When Eqs. (91–96) are substituted into Eq. (87), the solutions for the displacement and shear stresses in the layer can be obtained and expressed in the transform domain, as follows:

$$\begin{aligned} \bar{W}^{*(1)}(Y,\lambda,s) = \frac{\tau_0}{2s^2C_e^{(1)}\gamma_T^{(1)}}\Bigg\{ & \exp\left(-s\gamma_T^{(1)}|Y+H_P| - s\lambda H'\right) \\ & + \sum_{j=0}^{\infty}\Big[r_{1/0}^{j+1}r_{1/2}^{j}\exp\left(-s\gamma_T^{(1)}Y_{j1} - s\lambda H'\right) \\ & + r_{1/0}^{j}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j2} - s\lambda H'\right) \\ & + r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j3} - s\lambda H'\right) \\ & + r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j4} - s\lambda H\right)\Big]\Bigg\} \end{aligned} \tag{97}$$

$$\begin{aligned} \bar{\tau}_{YZ}^{*(1)}(Y,\lambda,s) = \frac{\tau_0}{2s}\Bigg\{ & (\pm)\exp\left(-s\gamma_T^{(1)}|Y+H_P| - s\lambda H'\right) \\ & + \sum_{j=0}^{\infty}\Big[r_{1/0}^{j+1}r_{1/2}^{j}\exp\left(-s\gamma_T^{(1)}Y_{j1} - s\lambda H'\right) \\ & - r_{1/0}^{j}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j2} - s\lambda H'\right) \\ & - r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j3} - s\lambda H'\right) \\ & + r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j4} - s\lambda H'\right)\Big]\Bigg\} \end{aligned} \tag{98}$$

$$\begin{aligned} \bar{\tau}_{XZ}^{*(1)}(Y,\lambda,s) = \frac{\tau_0\lambda}{2s\gamma_T^{(1)}}\Bigg\{ & \exp\left(-s\gamma_T^{(1)}|Y+H_P| - s\lambda H'\right) \\ & + \sum_{j=0}^{\infty}\Big[r_{1/0}^{j+1}r_{1/2}^{j}\exp\left(-s\gamma_T^{(1)}Y_{j1} - s\lambda H'\right) \\ & + r_{1/0}^{j}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j2} - s\lambda H'\right) \\ & + r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j3} - s\lambda H'\right) \\ & + r_{1/0}^{j+1}r_{1/2}^{j+1}\exp\left(-s\gamma_T^{(1)}Y_{j4} - s\lambda H'\right)\Big]\Bigg\} \end{aligned} \tag{99}$$

where

$$Y_{j1} = 2jH - (Y - H_P), \qquad Y_{j2} = 2(j+1)H + (Y - H_P)$$

$$Y_{j3} = 2(j+1)H + (Y + H_P), \qquad Y_{j4} = 2(j+1)H - (Y + H_P)$$

The sign $(\pm)$ expressed in Eq. (98) is chosen to be positive for $Y < -H_P$ and negative for $Y > -H_P$. By application of Cagniard's method[8] of Laplace inversion to Eqs. (98) and (99), the transient shear stresses can be obtained explicitly in the time domain. The final results are given in Appendix A.

Note that the solutions shown in Eqs. (A1–A4) are expressed in the transformed (X, Y) coordinate system. The transient solutions for the anisotropic layered half-place in the coordinate system of (x, y) can be obtained from Eqs. (32) and (33), in which the coordinate relations expressed in Eqs. (21–23) are used. Furthermore, the corresponding static solutions can be obtained by the application of the final value theorem to dynamic solutions in the Laplace transform domain. The results are given in Appendix B.

Numerical Results and Discussion

In the preceding section, the transient responses of shear stresses for an anisotropic layered half-space subjected to a dynamic antiplane concentrated loading have been analyzed. The corresponding numerical calculation will be carried out and discussed in detail in this section. The anisotropic layered half-space is subjected to a dynamic antiplane concentrated unit force with Heaviside function dependence that is located at $(x_0, y_0) = (0, -0.5h)$. It is known that the location of the applied loading will be shifted somewhere within the top layer after the linear coordinate transformation. The numerical results are divided into two parts. One is the case $b^{(1)} < b^{(2)}$, that is, the corresponding isotropic thin layer has a faster shear wave velocity than the half-space. The other is $b^{(1)} > b^{(2)}$, that is, the corresponding isotropic half-space has a faster shear-wave velocity. Because the transient solution is exact up to the arrival time of the next wave, only a finite number of waves will be involved in the numerical calculation. To give better insight into the transition behavior, the transient responses at the near field (horizontal position of the receiver is five times the thickness of the layer from the source) and the far field (200 times the thickness) will be considered in the following numerical evaluation. The locations of receivers are also limited within the top layer. The transient responses of shear stress τ_{xz} are not presented because of the similar characteristics to τ_{yz}. Note that the anisotropic static solutions proposed by Lin and Ma[33] are used for comparison with our static solutions, and the numerical results show a good match between these two different kinds of static solutions.

Case 1: $b^{(1)} < b^{(2)}$

Figures 5–7 show the transient shear stresses $\tau_{yz}^{(1)}$ for different vertical positions of receivers at the near field. It is found that the transient stress fields reveal a square-root singularity at the incident and reflected wave fronts because of the Heaviside time function dependence of the applied loading. In Figs. 5–7, the transient responses tend to corresponding static values uniformly after the first few waves passed the specified receiver. Note that the transient response approaches the corresponding static value slowly when the location of the receiver is near the free surface. Figures 8 and 9 show the transient responses at the near field $(x, y) = (5h, -0.5h)$ for $b^{(1)}/b^{(2)} = 1/5$ and 1/10, respectively. Note from Figs. 7–9 that the time needed to approach the static value for the transient solution will increase as the value of the ratio of $b^{(1)}/b^{(2)}$ decreases. The reason is that more reflected waves are induced for the case of small $b^{(1)}/b^{(2)}$. The transient shear stresses for $C_e^{(1)}/C_e^{(2)} = 5/1$ and 10/1 at the near field are plotted in Figs. 10 and 11, respectively. Note from Figs. 7, 10, and 11 that, the larger the magnitude of the ratio $C_e^{(1)}/C_e^{(2)}$, the longer the approaching time will be.

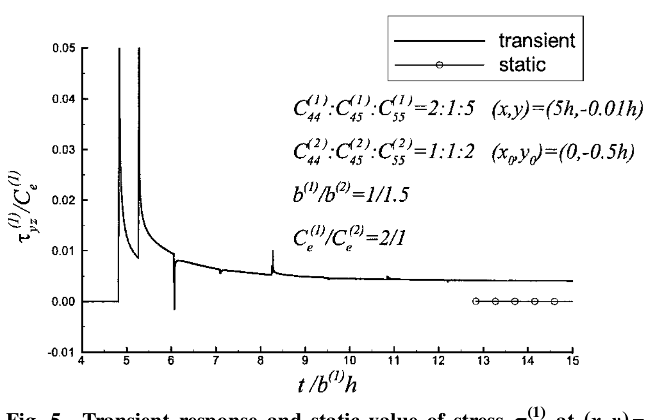

Fig. 5 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x, y) = (5h, -0.01h)$ in the layer with faster wave velocity.

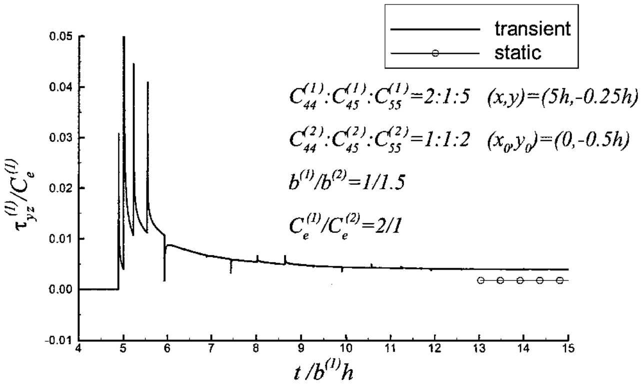

Fig. 6 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.25h)$ in the layer with faster wave velocity.

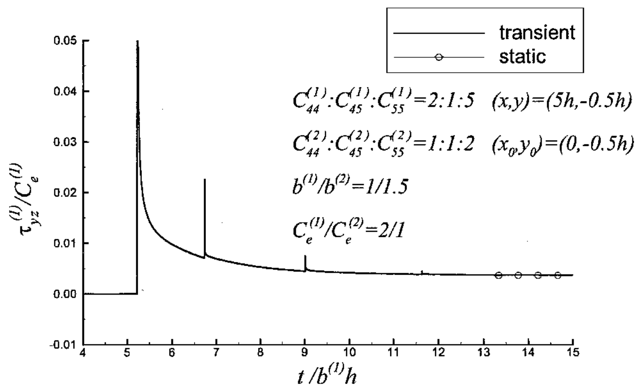

Fig. 7 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.5h)$ in the layer with faster wave velocity.

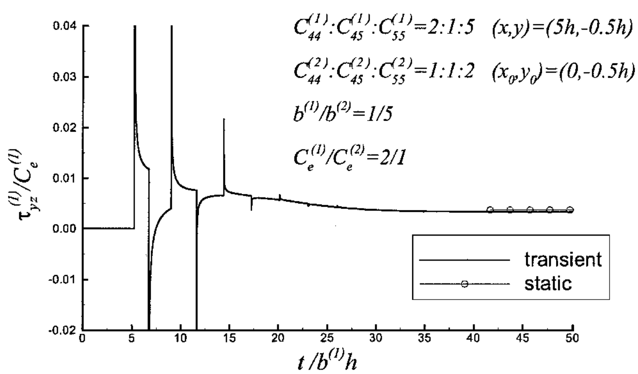

Fig. 8 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.5h)$ for $b^{(1)}/b^{(2)}=1/5$ in the layer with faster wave velocity.

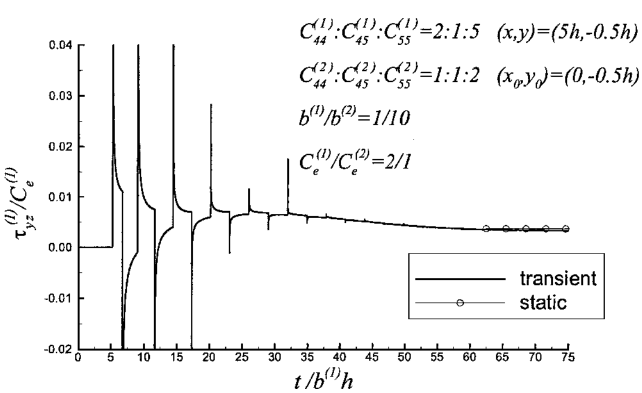

Fig. 9 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.5h)$ for $b^{(1)}/b^{(2)}=1/10.$ in the layer with faster wave velocity.

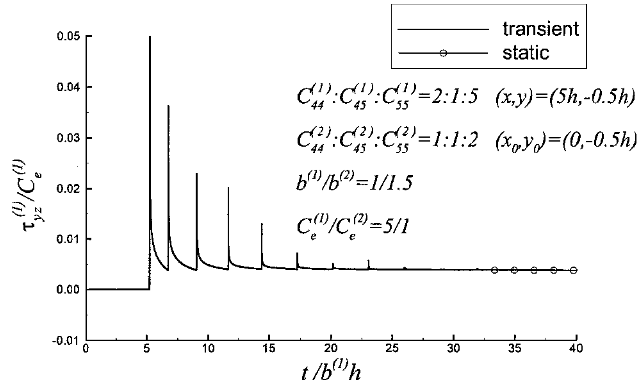

Fig. 10 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.5h)$ for $C_e^{(1)}/C_e^{(2)}=5/1$ in the layer with faster wave velocity.

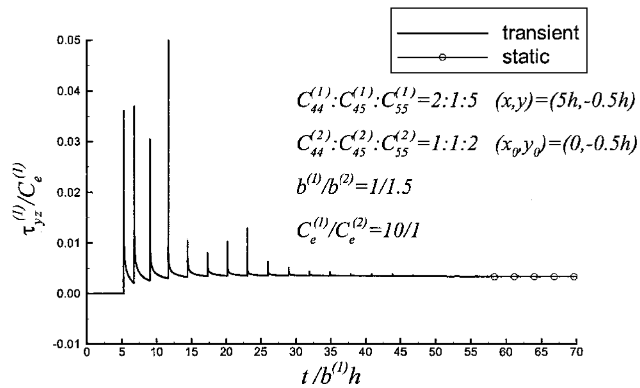

Fig. 11 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(5h,-0.5h)$ for $C_e^{(1)}/C_e^{(2)}=10/1$ in the layer with faster wave velocity.

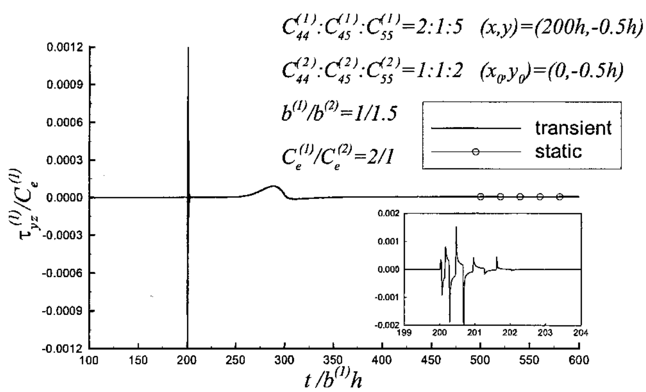

Fig. 12 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(200h,-0.5h)$ for $b^{(1)}/b^{(2)}=1/1.5$ in the layer with faster wave velocity.

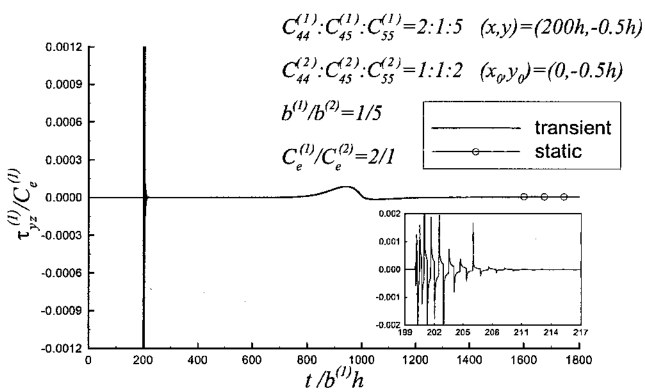

Fig. 13 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x,y)=(200h,-0.5h)$ for $b^{(1)}/b^{(2)}=1/5$ in the layer with faster wave velocity.

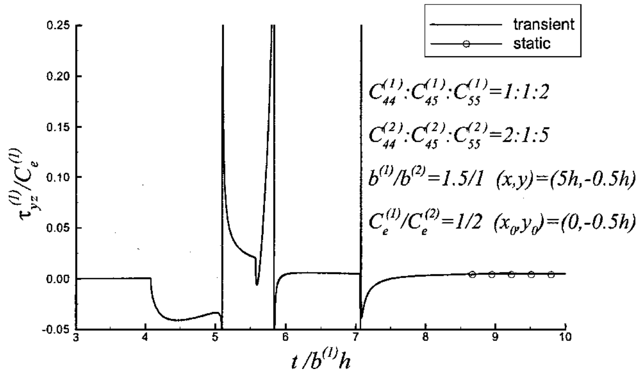

Fig. 14 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x, y) = (5h, -0.5h)$ for $b^{(1)}/b^{(2)} = 1.5/1$ in the layer with slower wave velocity.

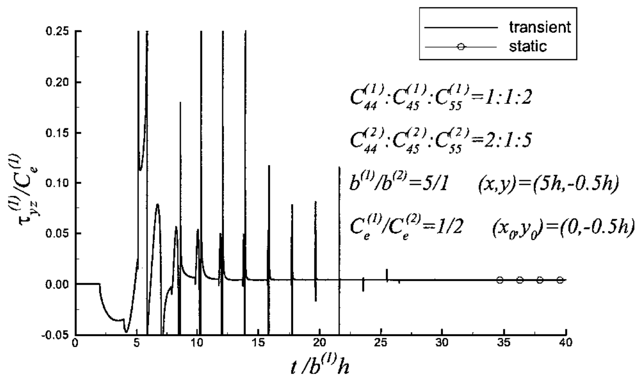

Fig. 15 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x, y) = (5h, -0.5h)$ for $b^{(1)}/b^{(2)} = 5/1$ in the layer with slower wave velocity.

The transient shear stresses $\tau_{yz}^{(1)}$ at the far field $(x, y) = (200h, -0.5h)$ for $b^{(1)}/b^{(2)} = 1/5$ and 1/5 are shown in Figs. 12 and 13, respectively. These results exhibit quite different and interesting phenomena from those of the near field. Note from Fig. 12 that the transient response varies significantly near the arrival of the incident wave (from $t/b^{(1)}h = 200$ to 202, as shown in the small window) and then approaches the static value smoothly for a period of time ($t/b^{(1)}h \approx 202 - 260$). This may mislead one to believe that transient response has become the static value already. However, the response will vary astonishingly ($t/b^{(1)}h \approx 260 - 300$) after this period and reaches a relative maximum value that is about 30 times the corresponding static value. Finally, the transient response then truly goes to the static value after the time $t \approx 300b^{(1)}h = 1.5xb^{(1)} = xb^{(2)}$. Note from Figs. 12 and 13 that the time for the transient solution to approach the static value will increase as the value of the ratio of $b^{(1)}/b^{(2)}$ decreases. If we calculate for many cases with different ratios of $b^{(1)}/b^{(2)}$, we find that the transient solution will truly tend to the corresponding static value only after time $t \approx xb^{(2)}$. This means that the transition from transient responses to static values for this case depends not only on the distance x between the source and receiver, but also on the slowness $b^{(2)}$ in the corresponding isotropic half-space. A similar result has also been found by Ma et al.[17] for the isotropic layered half-space problem.

Case 2: $b^{(1)} > b^{(2)}$

Figures 14 and 15 show the transient shear stresses $\tau_{yz}^{(1)}$ at the near field $(x, y) = (5h, -0.5h)$ for $b^{(1)}/b^{(2)} = 1.5/1$ and 5/1, respectively. Note that in Figs. 14 and 15, the first wave disturbing the receiver is a head wave, and the transient solution approaches the static value soon after all of the reflected head waves pass the receiver. Moreover, the number of reflected head waves passing the receiver will rise when the magnitude of the ratio $b^{(1)}/b^{(2)}$ is large. The transient response for $b^{(1)}/b^{(2)} = 1.5/1$ at the far field $(x, y) = (200h, -0.5h)$ is shown in Fig. 16. Note that the effect of reflected head waves plays a very significant role in Fig. 16. Because

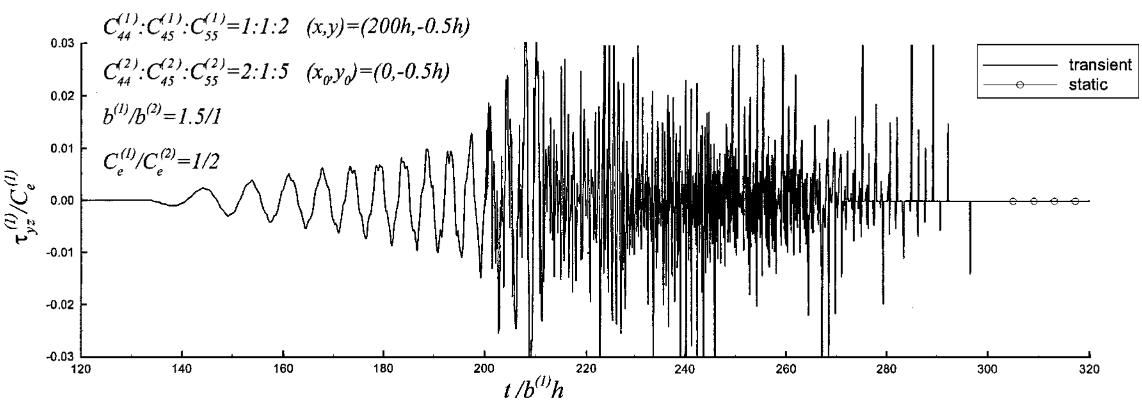

Fig. 16 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x, y) = (200h, -0.5h)$ for $b^{(1)}/b^{(2)} = 1.5/1$ in the layer with slower wave velocity.

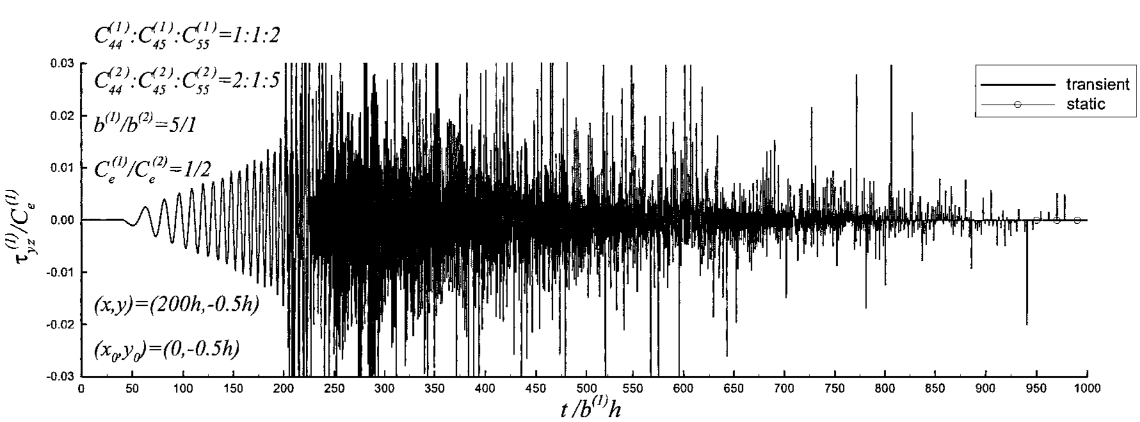

Fig. 17 Transient response and static value of stress $\tau_{yz}^{(1)}$ at $(x, y) = (200h, -0.5h)$ for $b^{(1)}/b^{(2)} = 5/1$ in the layer with slower wave velocity.

of the great number of reflected head waves, the transient response oscillates violently before it goes to the static value. It is also found that the transition time from transient solution to the static value is $t \approx 300 b^{(1)} h = 1.5 x b^{(1)} = x (b^{(1)})^2 / b^{(2)}$. To investigate this characteristic time in detail, the transient response for $b^{(1)}/b^{(2)} = 5/1$ at the far field $(x, y) = (200h, -0.5h)$ is shown in Fig. 17. It is indicated in Fig. 17 that the transient transition time extends longer as the value of the ratio of $b^{(1)}/b^{(2)}$ increases, and the characteristic time is still $t \approx 1000 b^{(1)} h = 5 x b^{(1)} = x (b^{(1)})^2 / b^{(2)}$. If we calculate for many cases with different ratios of $b^{(1)}/b^{(2)}$, we can conclude that the time needed for the transition from transient responses to static values for this case is $t \approx x (b^{(1)})^2 / b^{(2)}$, which depends on both wave velocities in the corresponding isotropic layer and half-space. This result is the same as that presented by Ma et al.[17] for an isotropic layered half-space.

Conclusions

In this study, the transient response of an anisotropic multilayered medium subjected to dynamic antiplane loadings is analyzed. A linear coordinate transformation is introduced to simplify the problem. The linear coordinate transformation reduces the anisotropic multilayered problem to an equivalent isotropic problem without complicating the geometry of the problem. An effective matrix method is used to derive the solutions in the Laplace transform domain. The solutions in the Laplace transform domain are constructed in the form of a power series of the phase-related reflection and transmission matrices. Each term in the series represents a reflected or transmitted wave. The transient solutions are then obtained by the Cagniard's method[8] of Laplace inversion. The corresponding static solution is also derived by application of the final value theorem. For numerical calculations, an anisotropic layered half-space is selected to investigate the transition from transient responses to static values in detail. It is indicated that the transition time for the case $b^{(1)} < b^{(2)}$ is $t = x b^{(2)}$, whereas for the case $b^{(1)} > b^{(2)}$, it is $t = x (b^{(1)})^2 / b^{(2)}$.

Appendix A: Transient Solutions in Time Domain

By application of Cagniard's method[8] of Laplace inversion to Eqs. (98) and (99), the transient shear stresses in the top layer for the case $b^{(2)} > b^{(1)} |\cos\theta_{jk}|$ can be expressed explicitly as follows:

$$
\begin{aligned}
\tau_{YZ}^{(1)}(X, Y, t) = \frac{\tau_0}{2\pi} \Bigg\{ & (\pm)\mathrm{Im}\left[\frac{\partial \lambda_0^+}{\partial t}\right] H(t - t_0) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j1}^+) r_{1/2}^{j}(\lambda_{j1}^+) \frac{\partial \lambda_{j1}^+}{\partial t}\right] H(t - t_{j1}) \\
& - \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j}(\lambda_{j2}^+) r_{1/2}^{j+1}(\lambda_{j2}^+) \frac{\partial \lambda_{j2}^+}{\partial t}\right] H(t - t_{j2}) \\
& - \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j3}^+) r_{1/2}^{j+1}(\lambda_{j3}^+) \frac{\partial \lambda_{j3}^+}{\partial t}\right] H(t - t_{j3}) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j4}^+) r_{1/2}^{j+1}(\lambda_{j4}^+) \frac{\partial \lambda_{j4}^+}{\partial t}\right] H(t - t_{j4}) \Bigg\}
\end{aligned}
\qquad \text{(A1)}
$$

$$
\begin{aligned}
\tau_{XZ}^{(1)}(X, Y, t) = \frac{\tau_0}{2\pi} \Bigg\{ & \mathrm{Im}\left[\frac{\lambda_0^+}{\gamma_T^{(1)}(\lambda_0^+)} \frac{\partial \lambda_0^+}{\partial t}\right] H(t - t_0) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j1}^+) r_{1/2}^{j}(\lambda_{j1}^+) \frac{\lambda_{j1}^+}{\gamma_T^{(1)}(\lambda_{j1}^+)} \frac{\partial \lambda_{j1}^+}{\partial t}\right] H(t - t_{j1}) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j}(\lambda_{j2}^+) r_{1/2}^{j+1}(\lambda_{j2}^+) \frac{\lambda_{j2}^+}{\gamma_T^{(1)}(\lambda_{j2}^+)} \frac{\partial \lambda_{j2}^+}{\partial t}\right] H(t - t_{j2}) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j3}^+) r_{1/2}^{j+1}(\lambda_{j3}^+) \frac{\lambda_{j3}^+}{\gamma_T^{(1)}(\lambda_{j3}^+)} \frac{\partial \lambda_{j3}^+}{\partial t}\right] H(t - t_{j3}) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{j4}^+) r_{1/2}^{j+1}(\lambda_{j4}^+) \frac{\lambda_{j4}^+}{\gamma_T^{(1)}(\lambda_{j4}^+)} \frac{\partial \lambda_{j4}^+}{\partial t}\right] H(t - t_{j4}) \Bigg\}
\end{aligned}
\qquad \text{(A2)}
$$

where

$$\lambda_0^+ = -(t/r_0)\cos\theta_0 + i|\sin\theta_0|\left(t^2 / r_0^2 - b^{(1)^2}\right)^{\frac{1}{2}}$$

$$r_0^2 = (X - H')^2 + (Y - H_P)^2, \qquad \theta_0 = \cos^{-1}[(X - H')/r_0]$$

$$\lambda_{jk}^+ = -(t/r_{jk})\cos\theta_{jk} + i|\sin\theta_{jk}|\left(t^2 / r_{jk}^2 - b^{(1)^2}\right)^{\frac{1}{2}}$$

$$r_{jk}^2 = (X - H')^2 + Y_{jk}^2, \qquad \theta_{jk} = \cos^{-1}[(X - H')/r_{jk}]$$

for $k = 1, 2, 3$, and 4, and where t_0 and t_{jk} are the arrival times of the stress waves.

For the case $b^{(2)} < b^{(1)} |\cos\theta_{jk}|$, the transient solutions in Eqs. (A1) and (A2) need to include an additional head wave contribution. They are given by

$$
\begin{aligned}
\tau_{YZ}^{h}(X, Y, t) = \frac{\tau_0}{2\pi} \Bigg\{ & \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{hj1}^+) r_{1/2}^{j}(\lambda_{hj1}^+) \frac{\partial \lambda_{hj1}^+}{\partial t}\right] \\
& \times H(t - t_{hj1}) H(t_{j1} - t) \\
& - \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j}(\lambda_{hj2}^+) r_{1/2}^{j+1}(\lambda_{hj2}^+) \frac{\partial \lambda_{hj2}^+}{\partial t}\right] H(t - t_{hj2}) H(t_{j2} - t) \\
& - \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{hj3}^+) r_{1/2}^{j+1}(\lambda_{hj3}^+) \frac{\partial \lambda_{hj3}^+}{\partial t}\right] \\
& \times H(t - t_{hj3}) H(t_{j3} - t) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{hj4}^+) r_{1/2}^{j+1}(\lambda_{hj4}^+) \frac{\partial \lambda_{hj4}^+}{\partial t}\right] \\
& \times H(t - t_{hj4}) H(t_{j4} - t) \Bigg\} H\left(|\cos(\theta_{jk})| - b^{(2)} / b^{(1)}\right)
\end{aligned}
\qquad \text{(A3)}
$$

$$
\begin{aligned}
\tau_{XZ}^{h}(X, Y, t) = \frac{\tau_0}{2\pi} \Bigg\{ & \sum_{j=0}^{\infty} \mathrm{Im}\Bigg[r_{1/0}^{j+1}(\lambda_{hj1}^+) r_{1/2}^{j}(\lambda_{hj1}^+) \\
& \times \frac{\lambda_{hj1}^+}{\gamma_T^{(1)}(\lambda_{hj1}^+)} \frac{\partial \lambda_{hj1}^+}{\partial t}\Bigg] H(t - t_{hj1}) H(t_{j1} - t) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j}(\lambda_{hj2}^+) r_{1/2}^{j+1}(\lambda_{hj2}^+) \frac{\lambda_{hj2}^+}{\gamma_T^{(1)}(\lambda_{hj2}^+)} \frac{\partial \lambda_{hj2}^+}{\partial t}\right] \\
& \times H(t - t_{hj2}) H(t_{j2} - t) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{hj3}^+) r_{1/2}^{j+1}(\lambda_{hj3}^+) \frac{\lambda_{hj3}^+}{\gamma_T^{(1)}(\lambda_{hj3}^+)} \frac{\partial \lambda_{hj3}^+}{\partial t}\right] \\
& \times H(t - t_{hj3}) H(t_{j3} - t) \\
& + \sum_{j=0}^{\infty} \mathrm{Im}\left[r_{1/0}^{j+1}(\lambda_{hj4}^+) r_{1/2}^{j+1}(\lambda_{hj4}^+) \frac{\lambda_{hj4}^+}{\gamma_T^{(1)}(\lambda_{hj4}^+)} \frac{\partial \lambda_{hj4}^+}{\partial t}\right] \\
& \times H(t - t_{hj4}) H(t_{j4} - t) \Bigg\} H\left(|\cos(\theta_{jk})| - b^{(2)} / b^{(1)}\right)
\end{aligned}
\qquad \text{(A4)}
$$

where t_{hjk} is the arrival time of the head wave, and

$$\lambda^{+}_{hjk} = -(t_{hjk}/r_{jk})\cos\theta_{jk} + |\sin\theta_{jk}|\left(b^{(1)^2} - t^2_{hjk}/r_{jk}\right)^{\frac{1}{2}} + i\varepsilon$$

$$t_{hjk} = \pm b^{(2)}(X - H') \pm \left(b^{(1)^2} - b^{(2)^2}\right)^{\frac{1}{2}} Y_{jk}$$

Appendix B: Static Solutions

The final value theorem states that

$$\lim_{t\to\infty} f(Y,t) = \lim_{s\to 0} s\bar{f}(Y,s) \tag{B1}$$

Thus, the static solutions can be deduced from the dynamic solutions expressed in Eqs. (98) and (99), and the final results are

$$\tau^{(1)}_{YZ}(X,Y) = \frac{\tau_0}{2\pi}\left[(\pm)\frac{|Y+H_P|}{(X-H')^2+(Y+H_P)^2} + \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j}_{1/2}\frac{Y_{j1}}{(X-H')^2+Y^2_{j1}} - \sum_{j=0}^{\infty} r'^{\,j}_{1/0} r'^{\,j+1}_{1/2}\frac{Y_{j2}}{(X-H')^2+Y^2_{j2}} - \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j+1}_{1/2}\frac{Y_{j3}}{(X-H')^2+Y^2_{j3}} + \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j+1}_{1/2}\frac{Y_{j4}}{(X-H')^2+Y^2_{j4}}\right] \tag{B2}$$

$$\tau^{(1)}_{YZ}(X,Y) = \frac{\tau_0}{2\pi}\left[\frac{(X-H')}{(x-H')^2+(Y+H_P)^2} + \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j}_{1/2}\frac{(X-H')}{(X-H')^2+Y^2_{j1}} + \sum_{j=0}^{\infty} r'^{\,j}_{1/0} r'^{\,j+1}_{1/2}\frac{(X-H')}{(X-H')^2+Y^2_{j2}} + \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j+1}_{1/2}\frac{(X-H')}{(X-H')^2+Y^2_{j3}} + \sum_{j=0}^{\infty} r'^{\,j+1}_{1/0} r'^{\,j+1}_{1/2}\frac{(X-H')}{(X-H')^2+Y^2_{j2}}\right] \tag{B3}$$

where $r'_{1/0} = 1$ and $r'_{1/2} = (C^{(1)}_e - C^{(2)}_e)/(C^{(1)}_e + C^{(2)}_e)$ are the static reflection coefficients.

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